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Newton, Goethe and the Mathematical Style of Thinking

/ Troy Vine

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Newton, Goethe and the Mathematical Style of Thinking

A Critique of Henri Bortoft's *Taking Appearance Seriously*

/ Troy Vine

I always carry Spinoza's Ethics with me; he brought mathematics into ethics, as I did into the science of colour [Farbenlehre]. That means there is nothing in the conclusion that is not grounded in the premise.[1]

- Goethe

1. Introduction

Johann Wolfgang von Goethe's approach to science is often characterized as holistic. The popularity of this characterization is due, in no small part, to Henri Bortoft's influential interpretation based on the idea of two kinds of unity.[2] Bortoft concludes *The Wholeness of Nature* by contrasting what he refers to as Goethe's "holistic" approach to science with the "analytical" approach of experimental science (328–330). In his most recent book, *Taking Appearance Seriously*, he contrasts the "concrete" nature of Goethean science with the "abstract" nature of experimental science. Experimental science is abstract because of the abstract nature of not only mathematics, but also of the mathematical style of thinking that is reflected in a twofold experimental method.

Bortoft's focus on the twofold method in experimental science, which proceeds from experience to theory and from theory back to experience, brings out an important aspect for understanding Goethe's approach to science and its relation to experimental science. I argue, however, that this shows not a difference between these two approaches,

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1. Goethe (2007, 621). My translation. Goethe made this comment in 1815 in conversation with the German art historian Sulpiz Boisserée.

2. For an historical account of these two kinds of unity in Kant and their development in Goethe, see Förster (2012).

but rather an important similarity which is often overlooked. The difference between the two approaches cannot therefore be characterized by Bortoft's distinction between concrete and abstract, which is based on a distinction between the mathematical and the hermeneutic styles of thinking. Using the example of colour, I show that this difference is best captured by the distinction between relations that are necessary and those that are contingent, which Bortoft calls "internal relations" and "external relations" respectively.

In section 2, I present Bortoft's account of the mathematical style of thinking in the history of experimental science, in which he distinguishes experimental method and the application of mathematics as two separate causes of abstraction.[3] In section 3, I compare Bortoft's example of exact sensory imagination with his example of geometrical proof. This shows that both examples are based on the idea of seeing internal relations. In section 4, I present the methodology of Newton's and Goethe's prism experiments. This shows that both Newton and Goethe use the twofold method and thus both exemplify the mathematical style of thinking. Bortoft's contrast, then, between and the abstract nature of the mathematical style of thinking and the concrete nature of Goethe's approach is mistaken. Then, in section 5, I compare Newton's and Goethe's prism experiments. This shows that the distinction between external and internal relations captures the difference between the two.

2. The Mathematical Style of Thinking in Experimental Science

In *Taking Appearance Seriously*, Bortoft identifies two kinds of unity with two kinds of thinking. The focus of the book is on what he calls the "hermeneutic style of thinking", in which "instead of the abstract universal of the mathematical style, we have the concrete universal" (168). The mathematical style of thinking is not just reflected in mathematics, but also in the twofold method employed in experimental science since its inception in the twelfth century. For Bortoft, the transition from the mathematical style of thinking to the hermeneutic style occurs in Goethe's approach to science, with Newton's and Goethe's prism experiments exemplifying this transition. In this section, we will consider Bortoft's historical overview of the twofold method in experimental science and his contrast between the abstract nature of the mathematical approach and the concrete nature of Goethe's approach.

Bortoft's historical account of experimental science is based on Alasdair C. Crombie's classic study *Robert Grosseteste and the Origins of Experimental Science*. Central to Crombie's study is the idea that "as in the thirteenth and fourteenth century, so in the later period, scientific method had two main aspects, the experimental and the mathematical" (296). Regarding the experimental aspect, Crombie states that:

> According to Aristotle, scientific investigation and explanation was a twofold process, the first inductive and the second deductive. The investigator must begin with what was prior in the order of knowing, that is, with facts observed through the senses, and he must ascend by induction to generalizations or universal forms or causes which were most remote from sensory experience, yet causing that experience and therefore prior in the order of nature. The second process in science was to descend again by deduction from these universal forms to the observed facts, which were thus explained by being demonstrated from prior and more general principles which were their cause. (25)

This twofold method is put forward in Aristotle's *Posterior Analytics*, which was rediscovered in the twelfth century and employed by scientists such as Robert Grosseteste

3. Bortoft also identifies a third cause of abstraction, namely mechanization. While I do not discuss mechanization in this essay, it is central to understanding not only the difference between Descartes and Newton, but also Goethe's critique of Newton's theory of colour.

and Roger Bacon in the thirteenth. They called the two stages "*resolutio*" and "*compositio*", which are Latin translations of the Greek "analysis" and "synthesis". Crombie comes to the conclusion that "the conception of the logical structure of experimental science held by such prominent leaders as Galileo, Francis Bacon, Descartes, and Newton was precisely that created in the thirteenth and fourteenth centuries" (3). As a result, "the history of the theory of experimental science from Grosseteste to Newton is in fact a set of variations on Aristotle's theme" (318).

The general principles determined by analysis—the first part of the twofold method—are not necessarily mathematical, and Francis Bacon gives "the most complete account of the non-mathematical side of the theory of experimental science" (300). Bortoft, too, includes Bacon among the scientists in whom "we find methodologically the same double procedure that had been developed since Grosseteste" (31), and thus follows Crombie in distinguishing between the application of the twofold method and the application of mathematics itself.

Bortoft calls the twofold method the "mathematical style of thinking" because, "although this double movement, from experience to theory and from theory to experience, is formulated by Aristotle expressly for science", it is "derived from the kind of reasoning which he observed being practised by the mathematicians (30).[4] The twofold method has "the effect of shifting attention away from the phenomenon" (31), with the result that "science becomes theory-centred instead of phenomenon-centred" (32). Moreover, "this is particularly the case when mathematics begins to play a fundamental role in science" and we "discover mathematical proportions and relationships in nature which lead us away from the diversity of sensory appearances towards the discovery of a unity which is more abstract" (32).

This idea of what Bortoft also calls an "abstract universal" led "to the remarkable idea that there are universal laws of nature" (32). As we have seen, Bortoft distinguishes between the application of the twofold method and of mathematics. Thus, while the abstract universal is an expression of the mathematical style of thinking, it is not necessarily itself mathematical. In a later passage on biology, for example, Bortoft contrasts the concrete universal of Goethe's archetypal plant (*Urpflanze*) with biologist Richard Owen's "abstract universal of a static generalisation" (83), which is a "minimal commonality from which all the specialised organs required by actual living organisms have been excluded" (84).

The distinction between the abstract nature of both mathematics and the mathematical style of thinking and the concrete nature of the sensory is a recurrent theme in Bortoft's book: "It is evident that, by its very nature, mathematics takes us away from the concrete into abstraction. But this in itself does not necessarily undermine the value of the sensory" (32). He also remarks that "although the mathematical style of thinking in physics leads us away from the experience of the senses as such, there is no intrinsic reason why this should make us think of the world as experienced through the senses as being inferior in any way to the relationships in nature discovered by means of mathematics" (32). While in the first passage Bortoft is contrasting mathematics itself with the sensory, in the second passage he is contrasting the twofold method with the sensory. For, as we have seen, we do not discover relationships in nature "by means of mathematics", but by means of the twofold method. This opposition between the sensory and both mathematics and the mathematical style of thinking shows that Bortoft considers both to be abstract by nature, in contrast to the concrete nature of the sensory.

Bortoft uses this distinction between the mathematical style of thinking and the sensory

4. I'm simplifying Bortoft's account slightly because he does not seem to be aware that Aristotle modelled what became the twofold method of *resolutio* and *compositio* on the twofold mathematical method of analysis and synthesis.

to contrast Goethe's approach to that of experimental science. He begins by stating that Goethe "returned to the senses and put sensory *experience* first instead of the mathematical" (53). And, in the context of plants, he remarks that:

> The movement of thinking here is indeed very different from looking for uniformities and commonalities in order to find a 'general plan common to all organs', which is the approach so often wrongly attributed to Goethe. The dynamic idea of the unity of nature that we find in Goethe is also very different from the kind of unity we find in the universal laws of nature, which came from the mathematical approach in science. (58)

In the second quote, Bortoft is contrasting two approaches to understanding plants, neither of which apply mathematics. So here he is using "the mathematical" and "the mathematical approach" to refer to the twofold method and the mathematical style of thinking. Thus, when Bortoft contrasts Goethe's "concrete" approach with the "abstract" mathematical approach, he is not merely contrasting Goethe's approach with mathematics, but with the mathematical style of thinking reflected in the experimental method. Characteristic of the mathematical style of thinking is the movement from phenomena to a general principle that is an abstract universal.

Bortoft claims that Goethe's approach to science is concrete compared to experimental science, which is abstract because it is based on the mathematical style of thinking. We will assess this claim by comparing, in the next section, Goethe's approach with mathematics, and then, in the following section, Goethe's approach with experimental science.

3. Exact Sensory Imagination and Mathematics

In the last section, we considered Bortoft's claim that Goethe's approach to science is concrete, whereas mathematics and the mathematical style of thinking reflected in the twofold method are abstract. In this section, we will assess this claim by juxtaposing Bortoft's example of exact sensory imagination with his example of geometrical proof.

Bortoft presents Goethe's method as having two stages, which he calls "active seeing" and "exact sensory imagination".[5] In the first stage, we put "attention into the sensory experience itself, entering into the lived experience of sensory perception, so that rather than just being 'sensory' in the empirical sense, it is better described as the 'sensuous' experience, or perception, of the phenomenon" (53). By becoming "aware of the sensuous quality of each colour", we transition from an empirical experience to sensuous experience (54-5). Bortoft says of Goethe that by "redirecting attention into sensuous experience he plunges into the sheer phenomenality of the phenomenon" (54).

In the second stage, we transition to a "sensuous-intuitive experience of phenomena" (53). This brings us "into contact with what is living, so that we begin to experience the phenomenon dynamically in its coming into being" (55). Bortoft gives the following description of exact sensory imagination:

> Now we put aside the physical manifestation and work entirely in imagination, trying to visualise what we have seen as exactly as we can. As we move through the colours at a boundary in imagination, we begin to experience their sensuous quality as if we were within the colours — one student described this as feeling like she was swimming through the colours. We find there is a dynamic quality in the colours at each boundary. What we experience is not separate colours — red, orange, yellow, or pale

5. Although the term "exact sensory imagination" is used by Goethe to designate the capacity for artistic activity, he does not relate it to scientific activity (1988, 46).

> blue, deeper blue, violet — but something more like 'red-lightening—to—orange—lightening—to—yellow' as a dynamic whole, and similarly with the darkening of blue to violet. There is a sense that the colours are different dynamic conditions of 'one' colour. This dynamic quality gives us an intuition of the wholeness of the colours at each boundary. This is not given directly to sense perception, but appears when sensuous perception is sublimed into intuition through the work of exact sensory imagination. In this way the sensuous-intuitive mode of perception replaces the verbal-intellectual mode. The colours are no longer thought of as being separate (verbal-intellectual) but are experienced as *belonging* together (sensuous-intuitive). The way to the wholeness of the phenomenon is through the doorway of the senses and not the intellectual mind. We find there is the sense of a *necessary* connection between the *qualities* of the colours at each boundary. It is not just accidental, for example, that the order of the colours is red, orange, yellow — and not red, yellow, orange — but is intrinsic to the colours themselves. (55–6)

When we transition from sensuous experience to sensuous-intuitive experience in exact sensory imagination, we see a necessary connection between colours—e.g. we see that orange *must* lie between red and yellow, not that it merely happens to do so. As Bortoft calls this necessary connection an "intrinsic relation" or "internal relation", the intuitive nature of sensuous-intuitive experience can be characterized as the seeing of internal relations.

Later in the book, Bortoft presents an example from mathematics to show "the difference between the mathematical and the empirical" (158). His gives the description of the proof that the sum of the interior angles of a triangle equals two right angles:

> A proof which would be mathematically acceptable would be one that did not involve measurement at all. It would be given entirely in terms of relationships between the angles without any need to refer to the actual size of the angles in a particular triangle. Consider any triangle ABC with angles *a*, *b*, and *c* (see Figure 1). Extend the side BC [Bortoft means AB] into a straight line, and draw a line through vertex A [Bortoft means C] parallel to this line (see Figure 2). Angle *a* equals angle *a*' because they are alternate angles between parallel lines. Angle *b* equals angle *b*' for the same reason. But angles *a*', *b*', and *c* must add up to 180° because they make a straight line. Hence it follows that angles *a*, *b*, and *c* must add up to 180°. In such a deductive proof we see that the angles of a triangle *must* add up to 180°. This is entirely different from just saying that the angles of a triangle do in fact add up to 180°. It's not that they happen to do so — as if this were an empirical discovery — but that they cannot not do so. (158)

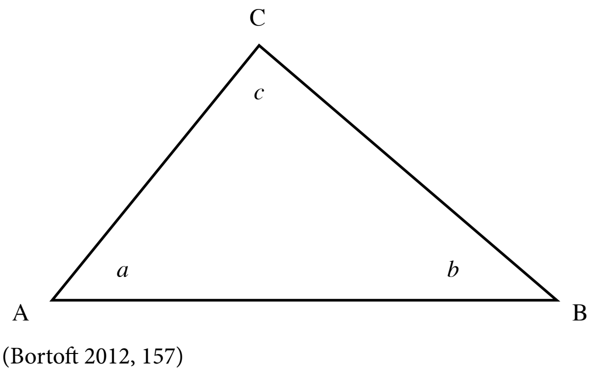

Figure 1. Triangle ABC. (Bortoft 2012, 157)

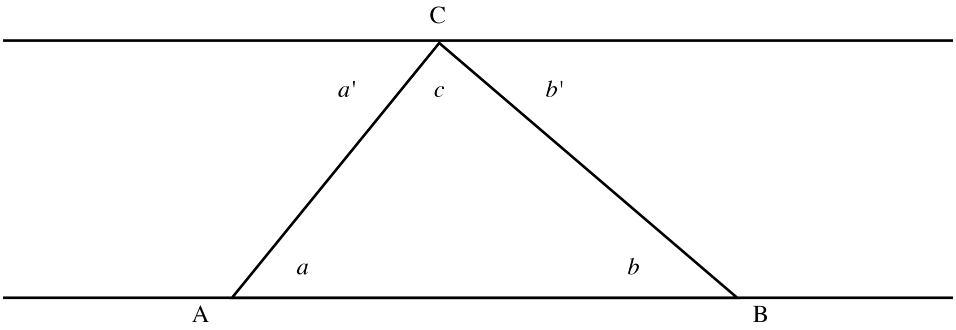

Figure 2. Triangle ABC with side AB extended and parallel line passing through vertex C. (Bortoft 2012, 157)

Central to this description is the distinction between a contingent relation between the angles, which we could determine empirically through measurement, and a necessary connection: we *see* that the angles of a triangle *must* add up to 180°, not that they merely happen to do so in this particular case. The nature of geometrical proof, then, is the seeing of internal relations. To complete the proof, we need to add the stage that allows us to see that alternate angles between parallel lines must be equal. To do this we can extend line AC to see that angle *a* must be equal to angle *a*'' (see figure 3). Then, by rotating the line AC about vertex C in our imagination, we can see that *a*'' must be equal to *a*' and thus that *a* must be equal to *a*'. Similarly for *b* and *b*'.

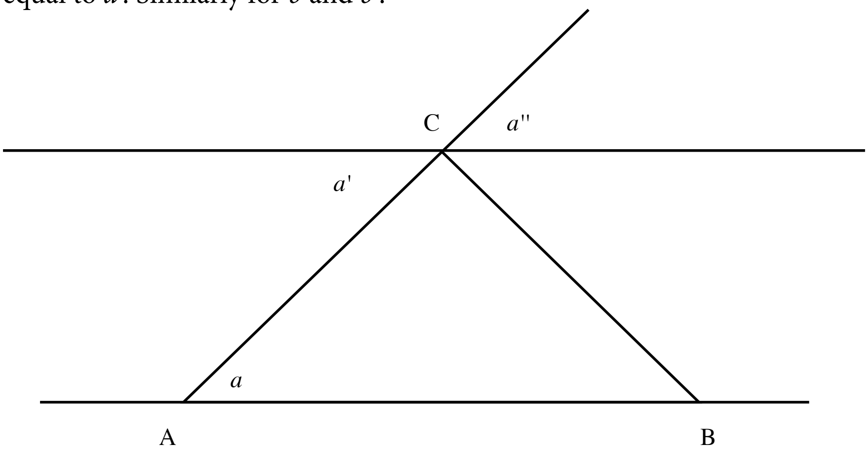

Figure 3. Triangle ABC with side AC extended to show that alternate angles between parallel lines are equal.

By juxtaposing Bortoft's example of exact sensory imagination and of mathematics (in the book they appear at the beginning and the end respectively), we can see what they have in common. In both examples, we begin with an empirical experience of an object containing contingently related parts: in the prism experiment, we see individual bands of colour produced by a prism, in the mathematical proof we see individual vertices. We then abstract the qualities in question in sensuous experience: in the prism experiments we abstract the individual colours from their "physical manifestation" (or "put aside" in Bortoft's euphemism); in the geometrical proof we abstract the lines that constitute the vertices. We then move between the different parts of an image: in the example of colour, we move between the different colours; in the example of the triangle we move between different vertices. Then, in a sensuous-intuitive experience, we see that the parts *must* be related in a certain way: we see necessary connections, or internal relations, that we did not see before.

In these two examples, there are no obvious criteria which allow us to apply the term "abstract" to one and "concrete" to the other. The only criterion for applying these terms would seem to be the movement from seeing external relations between empirical objects to seeing internal relations between properties of objects (i.e. primary and secondary qualities), which could be regarded as a process of abstraction. Bortoft remarks in a footnote that:

> Plato's achievement was to show that what is truly *mathematical* does not depend on working from sensory images of geometrical figures — for example, the discovery that the sum of the interior angles of a triangle is equal to two right angles (180°) does not depend on measuring the angles of drawn triangles, but follows directly from the very idea of a triangle. (184)

The comparison above, however, suggests a different account. In the example of colour, we work from a sensory image (we can consider exact sensory imagination to be sensory because we "visualise what we have seen as exactly as we can"). As Bortoft characterizes seeing internal relations not as an empirical experience but as a sensuous-intuitive experience, Plato's achievement could, therefore, be better described as showing a way of working from sensory images that is not empirical.

Plato's achievement, then, was to show that seeing external relations is distinct from seeing internal relations, or, to put the same point differently, that empirical experience is distinct from sensuous-intuitive experience. Even if it did "follow directly from the very idea of a triangle" that the sum of the interior angles is equal to two right angles, the idea itself consists of internal relations between geometrical elements (lines, points, etc.). The empirical diagram on the page is not the concept of a triangle, but it does represent the concept when we see the internal relations between the parts. The fact that Bortoft presents a proof with diagrams suggests that, rather than being an unnecessary detour, working from sensory images is necessary for grasping that very idea. The question, then, is not whether we need to work with sensory images in geometrical proofs, but how we work with them.

When we grasp the concrete unity of colours, "instead of abstracting unity from diversity, we have the intuition that the diversity is within unity" (57). Triangles, however, are an example of mathematics, and therefore, according to Bortoft, an abstract universal. Yet in the geometrical proof above we do not abstract unity from diversity—this would only be the case if we found empirically that the sum of the angles of a triangle are equal to two right angles. Rather, as with the example of colour, we can grasp the triangle "dynamically in its coming into being". This can be made clearer if we move vertex C of triangle ABC in our imagination: There is a sense that the triangles are different dynamic conditions of 'one' triangle (substituting "triangle" for "colour" into the description of exact sensory imagination above).

Let us consider the example of figure 4. How many triangles are there? If the concept triangle is an abstract universal—i.e. formed by abstracting what is common to the five figures—we must say that there are three triangles, because, as Bortoft notes, "all triangles are three-sided" polygons (218) and only three of the five figures have this property in common. However, if we grasp that the sum of the interior angles of a triangle is equal to two right angles we can say that there are five triangles: we can see the straight line AB as representing a triangle with one straight angle *c* and two zero angles *a* and *b*, as well as the line AB with two lines extended at right angles from the ends as represent a triangle with two right angles *a* and *b* and zero angle c. Moreover, we can see the five figures as five representations of one triangle—i.e. the concrete universal—dynamically in its coming into being.

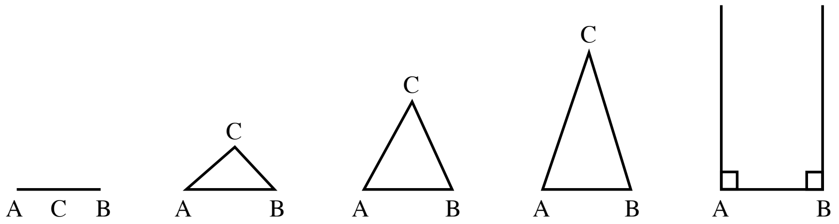

Figure 4. Three triangles ABC, five triangles ABC, or one triangle ABC dynamically coming into being.

Bortoft claims that in the mathematical style of thinking:

> every possible triangle is subsumed in advance under the universal concept 'triangle, of which any triangle is therefore a particular instance. Everything is included in the universal concept, so it is unthinkable that the universal itself could be enhanced by any *particular* triangle. The movement is only from the universal to the particular and never the other way round, so there simply cannot be any enhancement of the universal by the individual case. (125)

According to Euclidian geometry, a finite line AB with two lines extended at right angles from both its ends is not a triangle because parallel lines do not meet. But if we see an internal relation between this figure and a triangle—e.g. by extending the vertex C of the triangle in our imagination—, then we have an example of a particular triangle that enhances the universal. Thus, not only is an "enhancement of the universal by the individual case" conceivable, we have just done it!

The universal is enhanced, however, by abandoning the parallel postulate and thereby opening up the possibility for entirely new kinds of geometry. If we abandon the parallel postulate whilst retaining the possibility of measuring angles, there are two options. The first option is to retain the idea that the sum of the interior angles is equal to two right angles and extend Euclidean geometry to include ideal points. The second option is to allow the sum to be less than two right angles, which gives hyperbolic geometry, or greater than two right angles, which gives elliptic geometry (these two names are misleading because the triangle remains a three-sided polygon with straight sides in non-Euclidian geometry).

This example shows that Bortoft's static conception of mathematics is mistaken. Rather, as Michael Beaney and Robert Clark have shown, the idea of seeing internal relations "sheds light on the historical development of mathematical concepts" (2018, 133). This does not mean, however, that we must give up the distinction between the abstract and concrete universal; we just need to keep in mind that mathematical entities are not based on abstracting commonalities from empirical objects, and therefore not abstract universals.

Bortoft's examples of exact sensory imagination and geometrical proof show that there is no essential difference between them. His distinction between the abstract nature of mathematics and the concrete nature of exact sensory imagination is, then, a distinction without a difference. We will now turn to his contrast between Goethe's method and the experimental method.

4. Newton's and Goethe's Methodology

In section 2, we considered Bortoft's claim that modern science is abstract due to the abstract nature of not only mathematics, but also of the twofold method—i.e. the mathematical style of thinking. In section 3, we saw that mathematics is not by nature any more abstract than exact sensory imagination. A further problem for Bortoft's contrast between Goethe's concrete approach and the abstract approach of experimental science is created by Goethe's view of the relation of his method to that of mathematics. In a methodological reflection on his prism experiments, which we will consider in detail in the next section, Goethe describes sensuous-intuitive experience as follows:

> Such an experience, composed of many others, is clearly of a higher sort. It shows the general formula, so to speak, that overarches an array of individual arithmetic sums. In my view, it is the task of the scientific researcher to work toward experiences of this higher sort—and the example of the best men in the field supports this view. From the mathematician we must learn the meticulous care required to connect things in unbroken succession, or rather, to derive things step by step. Even where we do not venture to apply mathematics we must always work as though we had to satisfy the strictest of geometricians. (1988, 16)[6]

Bortoft's contrast between Goethe's approach to science and the mathematical style of thinking is therefore at odds with Goethe's own description of his method. In this section we will assess Bortoft's contrast by considering the methodology of Newton's and Goethe's prism experiments.

The most detailed description Newton gives of the methodology of his prism experiments is in an unpublished draft preface of the *Opticks*, which was first published in 1704:

> As Mathematicians have two Methods of doing things which they call Composition & Resolution & in all difficulties have recourse to their method of resolution before they compound so in explaining the Phaenomena of nature the like methods are to be used & he that expects success must resolve before he compounds. For the explications of Phaenomena are Problems much harder than those in Mathematicks. The method of Resolution consists in trying experiments & considering all the Phaenomena of nature relating to the subject in hand & drawing conclusions from them & examining the truth of those conclusions by new experiments & drawing new conclusions if it may be from those experiments & so proceeding alternately from experiments to conclusions & from conclusions to experiments untill you come to the general properties of things [& by experiments & phaenomena have established the truth of those properties]. Then assuming those properties as Principles of Philosophy you may by them explain the causes of such Phaenomena as follow from them: which is the method of Composition. (McGuire, 184–5)[7]

In this description, Newton is explicit about the relationship of his twofold method to the mathematical method of analysis and synthesis: analysis determines general principles; synthesis explains phenomena.

In the *Opticks*, Newton remarks that in the first book he "proceeded by this analysis to discover and prove the original Differences of the Rays of Light in respect of Refrangibility, Reflexibility, and Colour" (Newton, 405). He continues with the remarks that "these Discoveries being proved, may be assumed in the Method of Composition for explaining the Phaenomena arising from them" (Newton, 405). An example of composition is his explanation of the rainbow. Newton's prism experiments, then, are an example of the mathematical style of thinking in experimental science and a good example of Bortoft's distinction between the application of the twofold method and the application of mathematics. Mathematics can only be applied to colour once the general principle that equates colour with refrangibility has been determined by analysis. The analysis itself,

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6. I have modified Miller's translation to render the German "*Erfahrung*" as "experience", rather than "(piece of) empirical evidence".

7. The square brackets are Newton's additions. For an overview of the history of these passages and their relation to Newton's methodology, see Shapiro (2004).

then, is not the application of mathematics to colour, but the application of the mathematical method *by analogy* to determine general principles.

We will now turn to the method Goethe used in his prism experiments. A couple of months after publishing the second part of his *Contributions to Optics* in 1792, Goethe wrote a short methodological essay, quoted from above, which he later published with the title "The Experiment As Mediator between Object and Subject". In it, Goethe states that:

> My intention is to collect all the empirical evidence in this area, do every experiment myself, and develop the experiments in their most manifold variations so that they become easy to reproduce and more accessible. I will then attempt to establish the axioms in which the empirical evidence of a higher nature can be expressed, and see if these can be subsumed under still higher principles. (1988, 17)

Here we can discern three stages: experimentation (variation of the experiments), seeing internal relations (experience of a higher kind), and determining general principles. In a short methodological essay called "Empirical Observation and Science", written in 1798, these three stages are summarized under the rubric of "empirical phenomenon", "scientific phenomenon" and "pure phenomenon" (1988, 25). These correspond to Bortoft's three kinds of experience that we saw in section 3, namely empirical, sensuous, and sensuous-intuitive experience. However, while Bortoft describes sensuous experience as a plunge "into the sheer phenomenality of the phenomenon" (54), Goethe's describes scientific phenomena as seeing relationships that are "fully perceptible" (1988, 14-5). Bortoft's active seeing and exact sensory imagination are the transitions from empirical experience via sensual experience to sensuous-intuitive experience.

Goethe gives a methodological description in the *Didactic Part* of the *Farbenlehre*, published in 1810, in which he introduces the idea of an archetypal phenomenon as the general principle:

> In general, events we become aware of through experience are simply those we can categorize empirically after some observation. These empirical categories may be further subsumed under scientific categories leading to even higher levels. In the process we become familiar with certain requisite conditions for what is manifesting itself. From this point everything gradually falls into place under higher principles and laws revealed not to our reason through words and hypotheses, but to our intuitive perception through phenomena. We call these phenomena *archetypal phenomena* because nothing higher manifests itself in the world; such phenomena, on the other hand, make it possible for us to descend, just as we ascended, by going step by step from the archetypal phenomena to the most mundane occurrence in our daily experience. (§175/1988, 194-5)

Goethe's method proceeds from phenomena to general principles, which Goethe calls "archetypal phenomena" (*Urphänomene*). Once these general principles have been determined, they can be used to explain other phenomena. Goethe says that the principles show themselves not to reason, but to intuitive perception. In other words, the principles are seen, not merely thought. This corresponds to the idea of seeing connections and Bortoft's distinction between the verbal-intellectual and sensuous-intuitive mind.[8]

The similarity between Newton's and Goethe's methodological description is striking. Goethe is clearly using the twofold method in a manner similar to Newton. In a short essay called "Analysis and Synthesis", written in 1829, Goethe explicitly states that in his *Farbenlehre* he "used the analytic approach", which he characterized as presenting "every

8. For an account of Goethe's method in terms of Wittgenstein's idea of seeing connections, see Vine (2018).

known phenomenon in a certain sequence so that we could determine the degree to which all might be governed by a general principle" (1988, 48).

These passages show that Goethe is using the twofold method of analysis and synthesis in his prism experiments. Thus, both Newton and Goethe are using the mathematical method of analysis and synthesis by analogy. Goethe's method, like Newton's, is based on the mathematical style of thinking. Bortoft's contrast, then, between Goethe's approach to science and the mathematical style of thinking in experimental science is mistaken, and so we need another distinction to capture this difference. In the next section, we will turn to Bortoft's distinction between external and internal relations by considering Newton's and Goethe's prism experiments.

5. Newton's and Goethe's Prism Experiments

In Bortoft's description of exact sensory imagination in section 3, we saw that his account of Goethe's prism experiments is based on the idea of seeing internal relations between colours. Bortoft continues this description by comparing this approach to Newton's:

> This kind of connection between the qualities of the colours is missing from the Newtonian theory which asserts that light consists of colours which are separated when it is passed through a prism. In this case there is no intrinsic necessity in the order of the colours, but only an order that is imposed extrinsically by the attribution of a wavelength to each colour. (56)

Bortoft's contrast between Newton's and Goethe's account of colour is based on the distinction between external and internal relations. We will develop this idea by considering Newton's and Goethe's prism experiments.

In 1672, Newton published his "New Theory about Light and Colors" in the *Philosophical Transactions of the Royal Society*.[9] He begins by developing the following problem. Having allowed a narrow beam of sunlight to enter his darkened room through a small aperture in the window shutters and pass through a prism, he noticed that the image that fell on the wall opposite was coloured rather than white and five times longer than it was wide. He was able to calculate this difference to be much greater than Descartes' theory of refraction could account for. Thus, he was able to show not only that there was a hitherto undiscovered geometrical problem about light, but also that this problem was bound up with the problem of colour. By combining the geometrical problem with the chromatic problem in this way, a solution to the geometrical problem provides a solution to the chromatic problem. Moreover, the solution to the chromatic problem is in terms of geometry, rather than hypothetical corpuscles.

Newton's solution is his *experimentum crucis*. Placing a board with a small aperture just after the prism and a second about twelve feet away allowed him to select which part of the coloured spectrum passed through the two apertures by rotating the prism. His selection was refracted a second time by a prism placed behind the second aperture before it fell onto the wall opposite (see figure 5). Newton found that light from the violet end of the spectrum was refracted by a greater amount by the second prism than light from the red end. As the path of the light remained the same (it passed through the same two apertures), he came to his famous conclusion that "the true cause of the length of that [original prismatic] Image was detected to be no other, than that *Light* consists of *Rays differently refrangible*" (3079).

9. Cohen (1958, 47–59). The page number given for citations refers to the original publication of Newton's Letter.

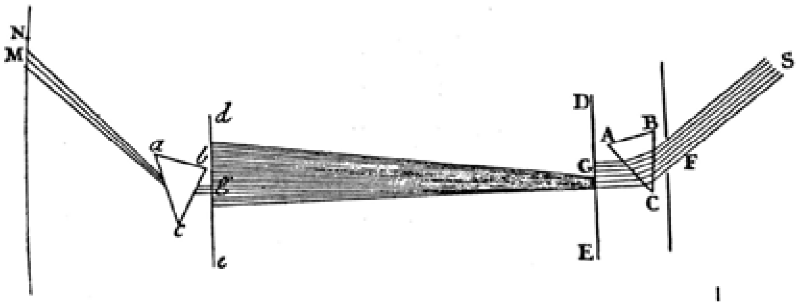

Figure 5. Newton's *experimentum* crucis. Light from the sun S passes through the aperture F in the window shutters and the prism ABC to produce a spectrum on board DE. Rays from a region of the spectrum are selected by aperture G in the board DE and aperture *g* in the second board *de*. The rays passing through both apertures pass through the second prism *abc* and fall on the screen at point M. Different regions of the spectrum are selected by rotating prism ABC (Newton, 47).

Newton uses the *experimentum crucis* to show that a geometrical property of a light ray—the amount by which it is refracted—is related to the colour it produces on the screen: "As the Rays of light differ in degree of Refrangibility, so they also differ in their disposition to exhibit this or that particular colour". Moreover, "to the same degrees of Refrangibility ever belongs the same colour", and "this Analogy 'twixt colours, and refrangibility, is very precise and strict" (3081). Thus, Newton solved the geometrical problem by showing that refrangibility is a property of light that is different for different kinds of rays, and solved the chromatic problem by equating colour with refrangibility. Newton managed, then, to give an account of colour in terms of refraction, and an account of refraction in terms of geometrical rays.

Newton's account of colour is a geometrical account: he uses his *experimentum crucis* to equate the colour caused by a ray with the ray's refrangibility. Nevertheless, this principle expresses an external relation of cause and effect, rather than an internal relation. As a result, we see *that* the degree of refraction is the cause of a particular colour, but we cannot see *why*: it does not show us that the rays that are refracted least *must* cause red. The order of the prismatic colours is therefore contingent, rather than necessary. In other words, it is conceivable that the order of the prismatic colours could be otherwise.

Newton's account of colour, then, leaves a gap in our understanding; for while we can see that refrangibility is the immediate (proximal) cause of colour, we can still ask for a further (remote) cause to explain why a given degree of refraction causes the particular colour that it does.[10] As a result, despite being a geometrical explanation rather than a mechanical explanation—in Newton's terminology a theory rather than an hypothesis—, it nevertheless opens the door to mechanical explanation. It is thus not surprising that beside the geometrical account of light and colour in the *Opticks* we find a mechanical account, although the two kinds of explanation are kept distinct.

We will now turn to Goethe's prismatic experiments. In 1791, Goethe published the first part of the *Contributions to Optics*. Despite being his first publication on colour, it contains his most perspicuous presentation of prism experiments. The presentation is based on observing black and white patterns through a prism.[11] It begins by showing that a homogenous white or black card seen through a prism remains unchanged (§41) and that a boundary between light and dark is necessary for prismatic colours to appear (§42).

This is followed by a number of complex forms presented that produce different colours, including black with white bands that produce Newton's spectrum (§44).
In order to "analyze these wonderful appearances", Goethe decomposes Newton's

10. For an account of proximal and remote causes in Newton's method see Ducheyne (2012, 18–47).

11. The first part of *Contributions to Optics* is in Goethe (1951, 6–37). My translations.

spectrum, which is produced by two boundaries between light and dark, into two spectra each of which is produced by a single boundary (§45). A horizontal boundary of black above white produces a distinct band of red above a distinct band of yellow, whereas a horizontal boundary of white above black produces a distinct band of blue above a distinct band of violet (§47-8). Goethe then provides a card with both situations next to each other so that the two edge spectra can be compared: blue appears opposite red and violet appears opposite yellow (see figure 6). This "shows that the colours do not follow one another, but "oppose one another" as "two opposing poles" (§§55, 72). Goethe has shown that the prismatic colours are governed by a principle of polarity.

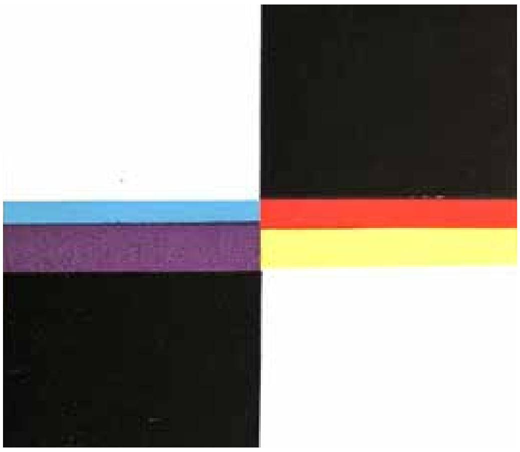

Figure 6. Goethe's illustration of the two edge spectra produced by looking at a boundary between black and white through a prism. *Contribution to Optics*, Card 14 (Goethe 1951, Plate VIII).

To produce Newton's spectrum, the two boundaries are brought together by viewing the black card with a horizontal white band first close up and then moving it further away. The two edge spectra separated by the white boundary come together, and green appears where the blue and yellow bands overlap (see figure 7). The next card is white with a horizontal black band, and the situation is reversed. This time magenta, or what Goethe calls "peach blossom", appears where the violet and red bands overlap (see figure 8). By moving the cards still further away, "the mixtures peach blossom and green [...] totally extinguish the colours of which they are composed" (§59).

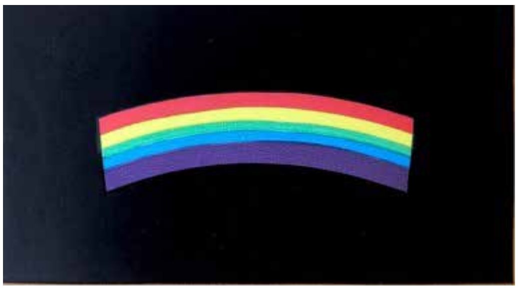

Figure 7. Goethe's illustration of the spectra produced by viewing a black card with a horizontal white band through a prism. Green appears where yellow and blue overlap. *Contributions to Optics*, Card 9 (Goethe 1951, Plate V).

Figure 8. Goethe's illustration of the spectra produced by viewing a white card with a horizontal black band through a prism. Goethe has not reproduced the magenta band that appears where violet and red overlap. *Contribution to Optics*, Card 10 (Goethe 1951, Plate V).

Goethe has demonstrated, then, that the Newtonian spectrum can be produced by combining two edge spectra. In addition to producing the familiar Newtonian spectrum, Goethe uses the principle of polarity to produce a complementary composite spectrum in which all the colours of the familiar Newtonian spectrum are replaced with their opposite, or complementary colour. Moreover, the inner two bands of the edge spectra, which overlap to produce green or magenta, disappear completely, leaving just three coloured bands: red, green and violet for the Newtonian spectrum; blue, magenta and yellow for the inverted spectrum. It thus appears as if the two inner bands of colour mixed to produce a new colour.

Bortoft suggests, as we saw in section 3, that the order of the prismatic colours "is intrinsic to the colours themselves" (56). We can apply this idea of internal relations between colours to understand Goethe's account of Newton's spectrum. As we have seen, Goethe's account of the two composite spectra is based on the idea of colour mixing. Therefore, there are two parts to an account based on an internal relation between colours. Firstly, we must show that there is an internal relation between the colours of the edge spectra, and, secondly, that there is an internal relation between the two colours that overlap and the colour they produce. As Bortoft's account presented in section 3 only addresses the first part, we have not yet seen that the colour produced when the two interior colours of the edge spectra overlap is internally related to the two interior colours.

Goethe's account is based on the polarity of light and dark. This refers to the polarity not only of white and black, but also of light colours and dark colours. We saw, in figure 6, not only that white is opposite black, but also that a light colour is opposite a dark colour. As the relation of light and dark is an internal relation, we can use it to see that Goethe's account is based on an internal relation between colours. We also saw that the edge spectra appear at a boundary between light and dark. They consist of a light colour (yellow or blue) next to white and a dark colour (red or violet) next to black. Thus, both edge spectra have the structure: white, light colour, dark colour, black. The one edge spectrum is therefore the opposite of the other in terms of the relation of light and dark. Thus, the internal relation that hold between white and black also holds between the two colours of the edge spectra: Yellow is lighter than red, blue lighter than violet; conversely, red is darker than yellow, violet darker than blue. Figure 6 shows that there is an internal relation between the colours of the edge spectra.

We have also seen that when two light colours overlap in the Newtonian spectrum, a dark colour is produced. Conversely, when two dark colours overlap in the complementary spectrum, a light colour is produced. Thus, two kinds of mixing occur, which are polar with respect to light and dark: a mixing that lightens two dark colours when they overlap, and a mixing that darkens two light colours when they overlap. Figures 7 and 8 show that there is an internal relation between the two colours that overlap and the colour they produce.

These two kinds of mixing are usually referred to as additive and subtractive mixing. However, this conception is misleading if one thinks of it as the addition or subtraction of coloured lights. An account in terms of rays would be a causal explanation, and therefore an empirical account based on external relations. Rather, when referring to a mixing of two colours, Goethe has in mind a mixing of the property of colour that is analogous to adding and subtracting the property of number. The statements "1 + 1 = 2" and "2 – 1 = 1" are not empirical statements about objects, but logical statements about numbers. Similarly, the statements about the mixing of colours by overlapping are not empirical statements about coloured lights, but logical statements about colours.[12]

We are now in a position to derive the form of the Newtonian spectrum from the polarity of light and dark. Because it is composed of two edge spectra whose light colours combine to produce a dark colour, it must consist of five coloured bands, starting with a dark colour and alternating between a light colour and a dark colour. We have thus given an account of the form of the Newtonian spectrum in terms of the internal relation of light and dark.

A comparison between Newton's and Goethe's prism experiments in terms of external and internal relations brings out an important difference: in Newton's approach, we are able to see a relation between an angle of refraction and a particular colour, but it is only in Goethe's that we are able to see an internal relation between the colours themselves. For Newton, the order of the colours appears contingent, but Goethe shows that it is necessary. Put another way, in Newton's theory it is conceivable that the order of the prismatic colours could be different, in Goethe's it is not. Thus, despite Newton's approach being geometrical, this comparison shows that Goethe's approach is closer to mathematics, because it is based on internal relations.

In the last section we saw that both Newton and Goethe use the twofold method to determine general principles, and then use these principles to explain other phenomena. In this section we have seen that Newton uses analysis to determine an external relation between the refrangibility of a light ray and the colour it causes, and then uses synthesis to explain why sunlight creates a coloured image when passed through a prism. Goethe, on the other hand, uses an analysis of prism experiments to determine internal relations between prismatic colours, which are expressed in the principle of polarity. He then uses synthesis to explain the composition of Newton's spectrum. As in the last two sections, Bortoft's distinction between concrete and abstract seems inapplicable here. Bortoft's distinction between external and internal relations, on the other hand, brings out an essential difference between Newton's and Goethe's account of colour.

6. Conclusion

We began by considering Bortoft's account of the history of experimental science and the supposed abstraction due both to the application of mathematics and to the mathematical style of thinking reflected in the twofold method. Juxtaposing Bortoft's example of exact sensory imagination with his example of geometrical proof showed that the difference between them cannot be captured by the contrast of concrete and abstract, because both examples are based on seeing internal relations between properties. Similarly, our juxtaposition of Newton's and Goethe's methodological descriptions showed that both Newton and Goethe used analysis to determine general principles and synthesis to explain phenomena. Goethe's approach cannot, therefore, be considered concrete in comparison to a supposedly abstract mathematical style of thinking.

12. For an account of additive and subtractive mixing in relation to Goethe's prism experiments, see Wilson (2018).

Our comparison of Newton's and Goethe's prism experiments, on the other hand, demonstrated that while Newton shows that there is an external relation between prismatic colours and refrangibility, Goethe shows that there is an internal relation between the prismatic colours themselves. Goethe demonstrated, then, that the order of the prismatic colour is not contingent, but necessary. Our investigation shows that Boftoft's distinction between internal and external relations is able to capture the difference between Newton's and Goethe's approach to colour. In addition, it shows that rather than representing a break with experimental science, Goethe's approach to science expresses the diversity within the unity of the mathematical style of thinking.

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